# *The Black Hole Information Paradox, Complementarity or Firewalls?*

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SUT

# Particle Creation by Black Holes (Hawking 1975)

- Treating gravity classically, while matter fields are quantum mechanical
- The concept of particles becomes frame dependent in curved space-times.

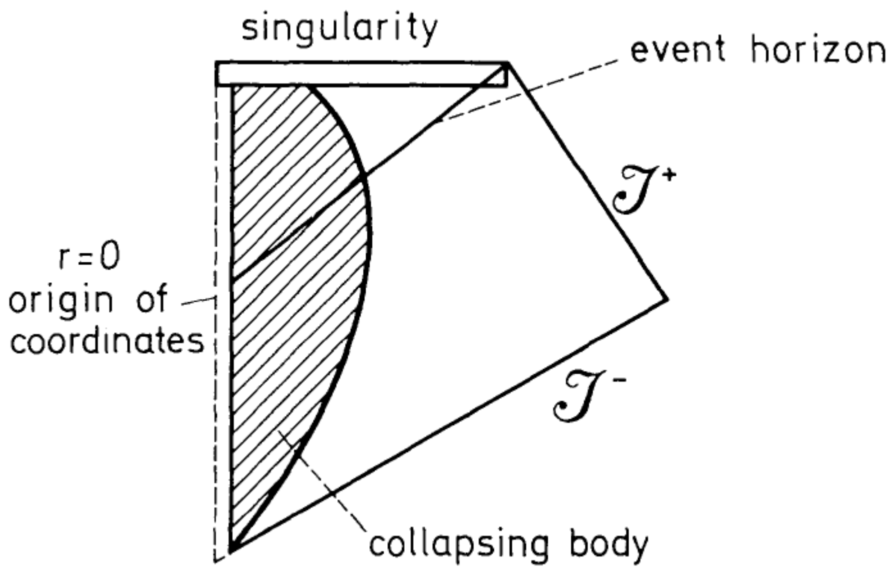
$$\Phi = \sum_i \left(f_i a_i + f_i^* a_i^\dagger\right) = \sum_i \left(p_i b_i + p_i^* b_i^\dagger\right)$$

$$p_i = \sum_j \alpha_{ij} f_j + \beta_{ij} f_j^* \Rightarrow \langle 0|_a \left(N_i^b\right) |0\rangle_a = \sum_j \left|\beta_{ij}\right|^2$$

# Particle Creation by Black Holes (Hawking 1975)

- Hawking did the Calculations for a collapsing star.
- Empty space in the far past, $I^{-}$ in vacuum states $|0\rangle_{in}$, would contain a thermal flux of *out*-particles:

$$\langle 0|_{in}\left(N_i^{out}\right)|0\rangle_{in} = \frac{\Gamma_i}{e^{\frac{2\omega_i\pi}{\kappa}} - 1}.$$




From Hawking 1975

# Particle Creation by Black Holes (Hawking 1975)

- Black holes are Really thermal: $T = \hbar \frac{\kappa}{2\pi}$, $\mathrm{S} = \frac{\mathrm{c}^3 \mathrm{A}}{4\mathrm{G}\hbar}$

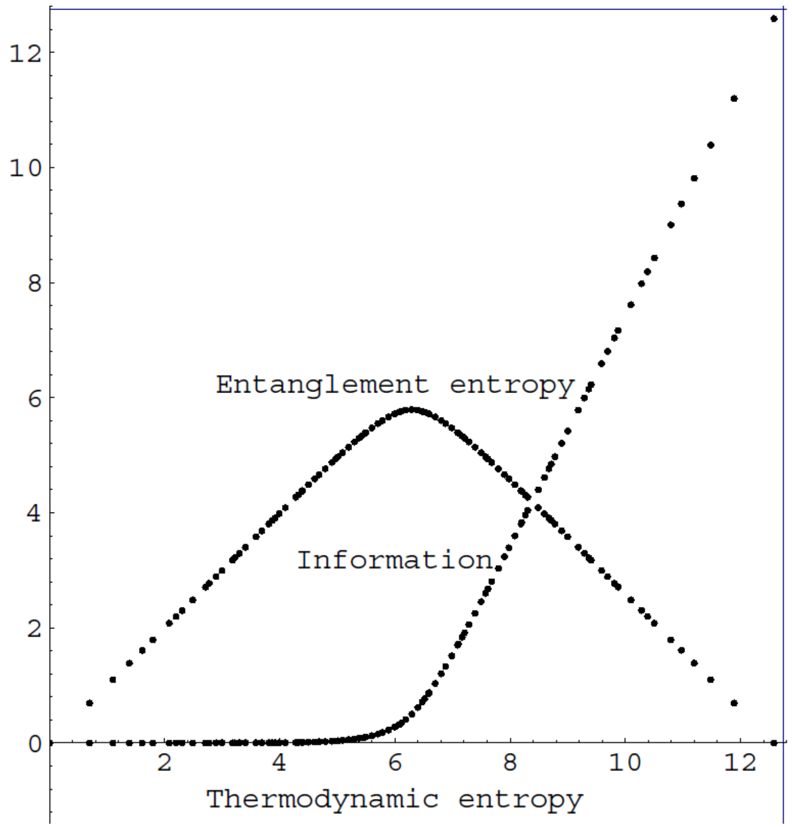
- Black holes Evaporate: $\frac{dE}{dt} \approx -\alpha A T^4 \xrightarrow[E=M, A\propto M^2]{} \tau \sim M^3$

# Page Curves (Page 1993)

- Page assumed that the black hole and radiation to be in pure state in an *mn* dimensional Hilbert space.
- He plotted the average entanglement entropy and information of the radiation:

$$S_r = -tr_r(\rho_r ln\rho_r), \qquad I_r = ln\, m - S_r$$

# Page Curves (Page 1993)




From Page 1993

# Page's Theorem

- More precisely we can explain the above statement in the following theorem.
- If we choose a random state by scrambling a given state using a random unitary matrix:

$$|\psi(U)\rangle \equiv U|\psi_0\rangle$$

- *Page's Theorem: For any bipartite space $H_A \otimes H_B$, one has:*

$$\int dU \left\| \rho_A(U) - \frac{I_A}{|A|} \right\|_1 \leq \sqrt{\frac{|A|^2 - 1}{|A||B| + 1}}$$

# Page's Theorem

- Sketch of the proof:

$$\left(\int dU \left\|\rho_A(U) - \frac{I_A}{|A|}\right\|_1\right)^2 \leq \int dU \left(\left\|\rho_A(U) - \frac{I_A}{|A|}\right\|_1\right)^2 \leq |A| \int dU \left(\left\|\rho_A(U) - \frac{I_A}{|A|}\right\|_2\right)^2$$

- A straightforward calculation shows:

$$\left(\left\|\rho_A(U) - \frac{I_A}{|A|}\right\|_2\right)^2 = \mathrm{tr}\left(\left(\rho_A(U) - \frac{I_A}{|A|}\right)^\dagger \left(\rho_A(U) - \frac{I_A}{|A|}\right)\right) = tr((\rho_A(U)^2) - \frac{1}{|A|}$$

- Then we plug in the above expression and perform the integration. Then by taking the square root, result follows.

# Page's Theorem

- We have the thermodynamic entropies:

$$S_R^{\{course\}} = \log|R|, \qquad S_{BH}^{\{course\}} = \log|BH|$$

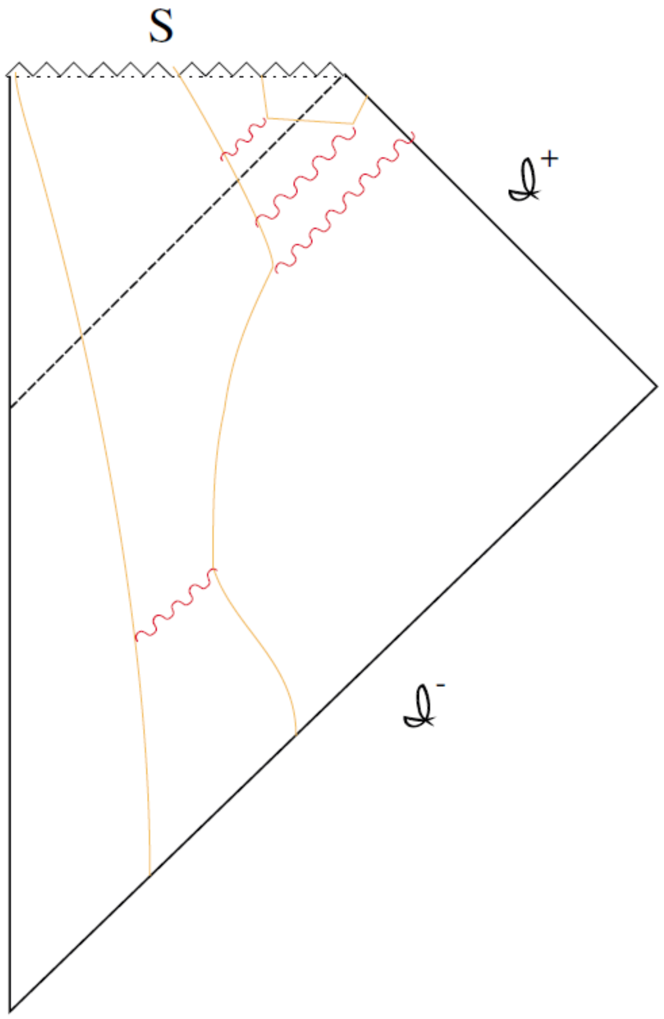
- For photon gas: $S_R^{\{course\}} \propto tT$
- For black hole: $S_{BH}^{\{course\}} \propto M^2, \qquad for\; t \ll M^3$

- Thus by page's theorem: $S_R \approx S_R^{\{course\}}, \qquad before\;\; t_{page}$
- After $t_{page}$, we would have $S_{BH} \approx S_{BH}^{\{course\}}$ and thus: $S_R \approx S_{BH}^{\{course\}}$

# The Information Paradox

- In 1976 Hawking argued that the black hole evaporation would violate information conservation or more precisely Unitarity.
- From Unitarity one expects: $|\psi_{out}\rangle = S|\psi_{in}\rangle$

# The Information Paradox




From Susskind 2004

# The Information Paradox

- Hawking relied on locality to claim:

$$H_{in} = H_{I^-}, \qquad H_{out} = H_{I^+} \otimes H_S$$

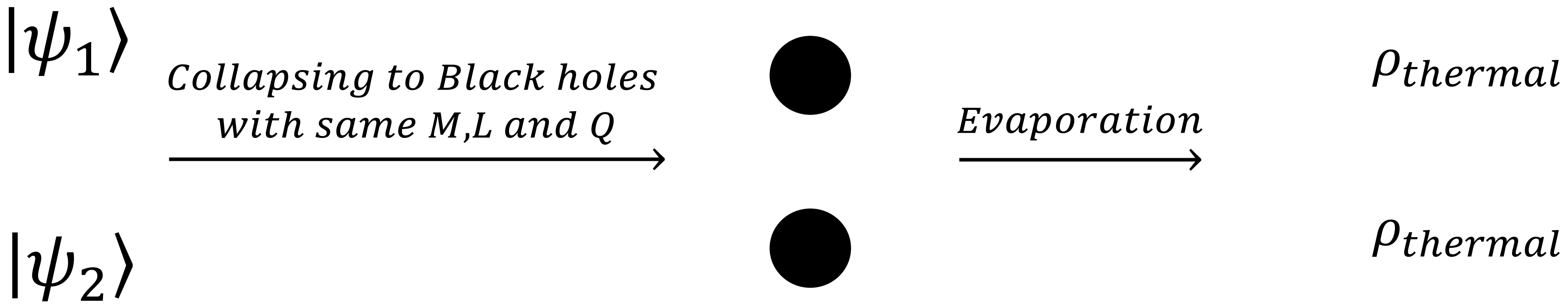
- From the point of view of the outside observer at $I^+$:

$$\rho_{out} = tr_S |\psi_{out}\rangle\langle\psi_{out}|$$

- Hawking claimed that the purity would not be restored if the black hole evaporated.

# The Information Paradox

- An example:

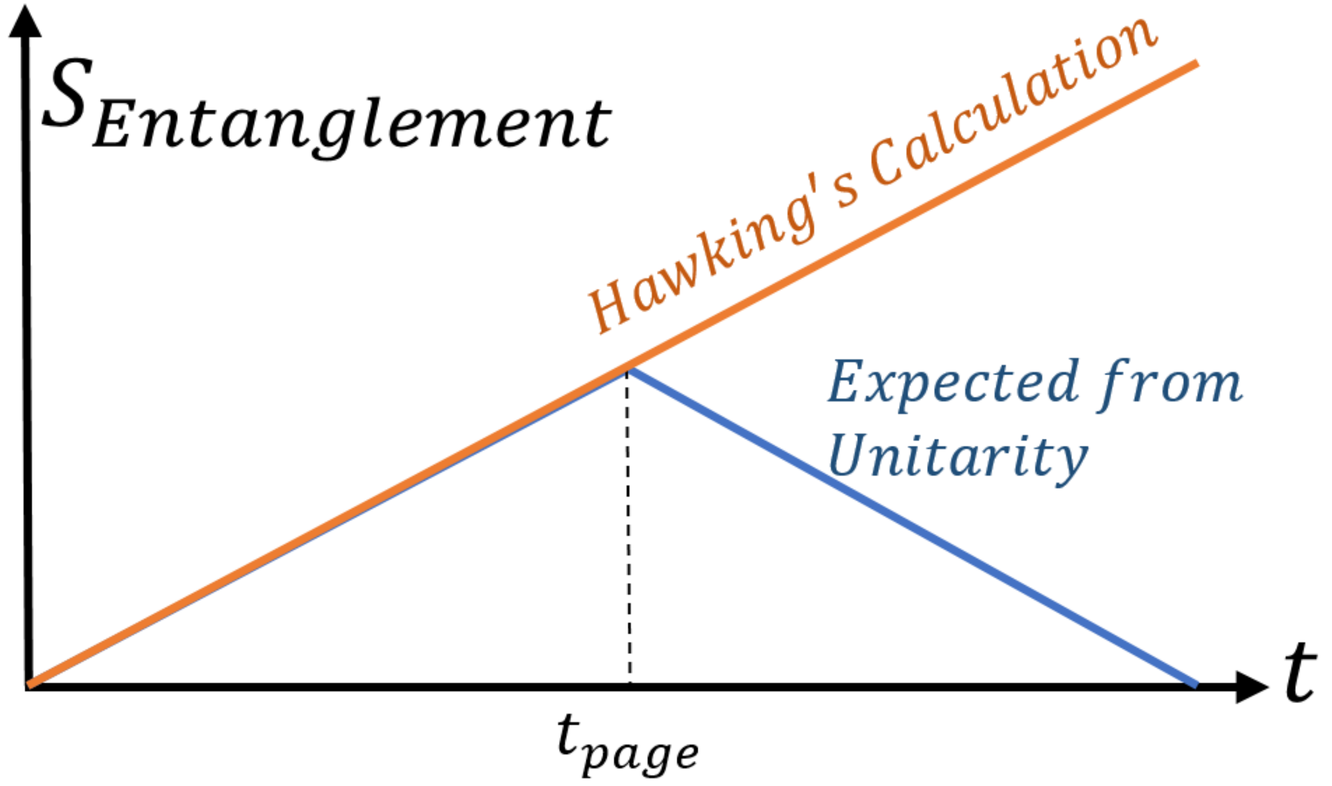


# The Information Paradox

- There may be two possibilities:
  1- Information is lost.
  2- Information is restored at the very end where the quantum gravity effects are important.
  3- The black hole does not fully evaporate. At the very end there would be some Planckian "remnant" that is entangled with the radiation.

# The Information Paradox

- The 2nd possibility would still violate unitarity:



# Some naive resolutions

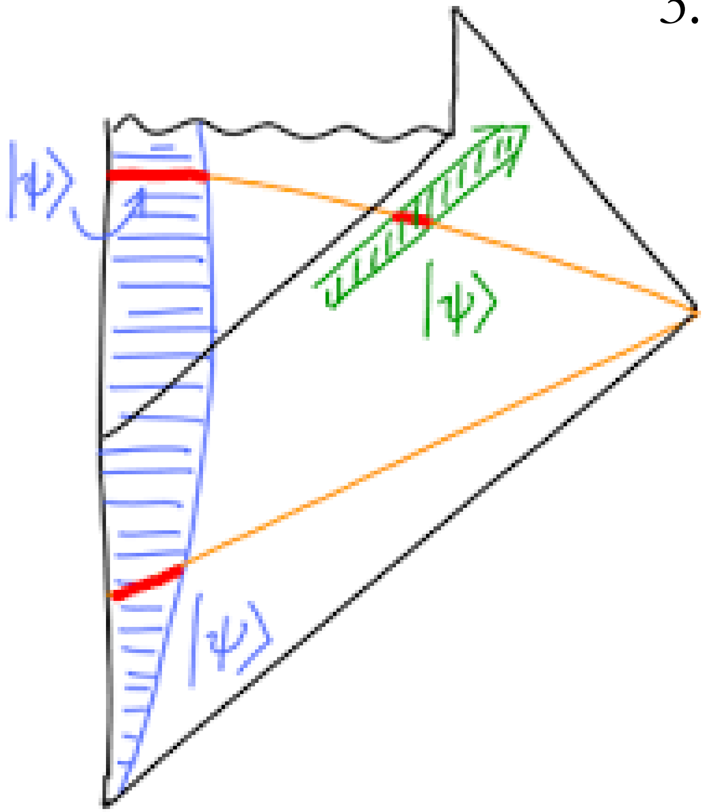
1. The horizon is impregnable for in-falling observers.→ Violates EP
2. What if information is both returned out by some mechanism and also passed freely through the horizon. → Violates the no-cloning theorem (Xeroxing Paradox)

# Some naive resolutions

- Xeroxing Paradox:
  1. A state $|\psi\rangle$ is thrown into the black hole.
  2. By unitarity the Hawking cloud would be in the state $|\psi\rangle$.
  3. By EP state $|\psi\rangle$ passes through the horizon.

  Thus one has:

$$|\psi\rangle \to |\psi\rangle \otimes |\psi\rangle$$



# Remark: Stretched Horizon and exterior

- We recall in a simple scalar field model in a black hole background:

$$E = \sum_i \frac{\Gamma_i \omega_i}{e^{\frac{2\omega_i \pi}{\kappa}} - 1}.$$

- What are the $\omega_i$s?! And what separates the different modes?!
- We can mend this ambiguity by a UV cutoff near the horizon by simply putting Dirichet b.c. $\phi = 0$ on a surface a Planckian distance from horizon.
- There is also an IR divergence for $r \to \infty$, which could be mended by a barrier at some large $r$ outside the horizon.

# Remark: Stretched Horizon and exterior

- One can use a simplified model of scalar field theory in rindler space-time. The for each Fourier mode one has (see Susskind 2004):

$$-\frac{\partial^2 \chi_k}{\partial u^2} + (k^2 e^{2u})\chi_k = \lambda^2 \chi_k$$

- The IR cutoff is provided by the potential, $V(u) = k^2 \exp(2u)$, Which is large for $u > -logk$. For simplification we may approximate this potential by Dirichet b.c. at $u = u_1 = -logk$.

# Remark: Stretched Horizon and exterior

- In order to specify the UV cutoff, one can introduce a cutoff at $u_0 = \log \epsilon$ for field or its derivative to vanish.
  $\epsilon$ is a distance from the horizon. Then one would remove this cutoff by $\epsilon \to 0$.
- Thus for a given $\epsilon$ and $k$, we have a field theory inside a box of length:

$$L(k) = u_1 - u_0 = -\log(\epsilon k)$$

# Remark: Stretched Horizon and exterior

- We then expand for each $k$ the field $\chi_k$ and the total density matrix would be given by: (more on this later)

$$\rho_R = \prod_{n,k} \rho_R(n,k)\,, \qquad \rho_R(n,k) \sim \exp\left(-2\pi\lambda(n,k)a^\dagger(n,k)a(n,k)\right)$$

- A calculation similar to that of Hawking would give:

$$\langle N(n,k)\rangle = \frac{1}{\exp\big(2\pi\lambda(n,k)\big) - 1}$$

- These particles constitute a so called "thermal atmosphere" outside the horizon.

# Remark: Stretched Horizon and exterior

- As mentioned in the previous slide and will be shown later, vacuum state in each wedge is a thermal state, so calculation of the entanglement entropy reduces to thermodynamical methods.

- For Bosonic gas in 1+1 dim: $\frac{S}{\mathrm{L}} = \frac{\pi}{3} T \xrightarrow{T=\frac{1}{2\pi},\ L(k)=-\log(k\epsilon)} S(k) = \frac{1}{6}|\log(k\epsilon)|$

- Then: $S_{tot} = \frac{A}{24\pi^2}\int d^2k|\log(k\epsilon)| \approx \frac{1}{96\pi^2}\frac{A}{\epsilon^2}$

# Remark: Stretched Horizon and exterior

- Thus we arrived at:

$$S \approx \frac{A}{\epsilon^2}$$

# Remark: Stretched Horizon and exterior

- This entropy diverges at $\epsilon \to 0$.
- If ordinary quantum field theory is to hold up to some distance outside the horizon, then it must not exceed the Bekenstein-Hawking entropy. Thus:

$$\epsilon^2 \leq \frac{G\hbar}{c^3} = l_p^2$$

- The above calculation, Motivates the definition of an effective membrane, or "stretched" horizon at a distance of roughly $l_p$ from the mathematical horizon.

# Remark: Stretched Horizon and exterior

- In fact in this model we have essentially replaced the black hole with the field theory degrees of freedom near the horizon, and there is nothing left for the black hole to do.
- This choice $\epsilon$ of was rather arbitrary, and by making it larger we can imagine a separation of degrees of freedom into QFT degrees of freedom at distances greater than $\epsilon$ from the horizon and "quantum gravity" degrees of freedom closer in.
- In this picture the stretched horizon is in thermal equilibrium with the QFT modes in the atmosphere, and evaporation happens because these modes occasionally tunnel out to infinity, because the real black hole's repulsive potential is not a perfect wall.

# Remark: Stretched Horizon and exterior

- With the model discussed in the former pages, we might want to argue that to an outside observer physics is essentially described by knowledge of 3 regions: Stretched horizon, Atmosphere and the radiation outside the potential barrier. (More on this later.)

# Black Hole Complementarity

- To remedy the Xeroxing paradox, Complementarity in it's simplest forms states that no observer can see both copies so it does not matter that Xeroxing happens.

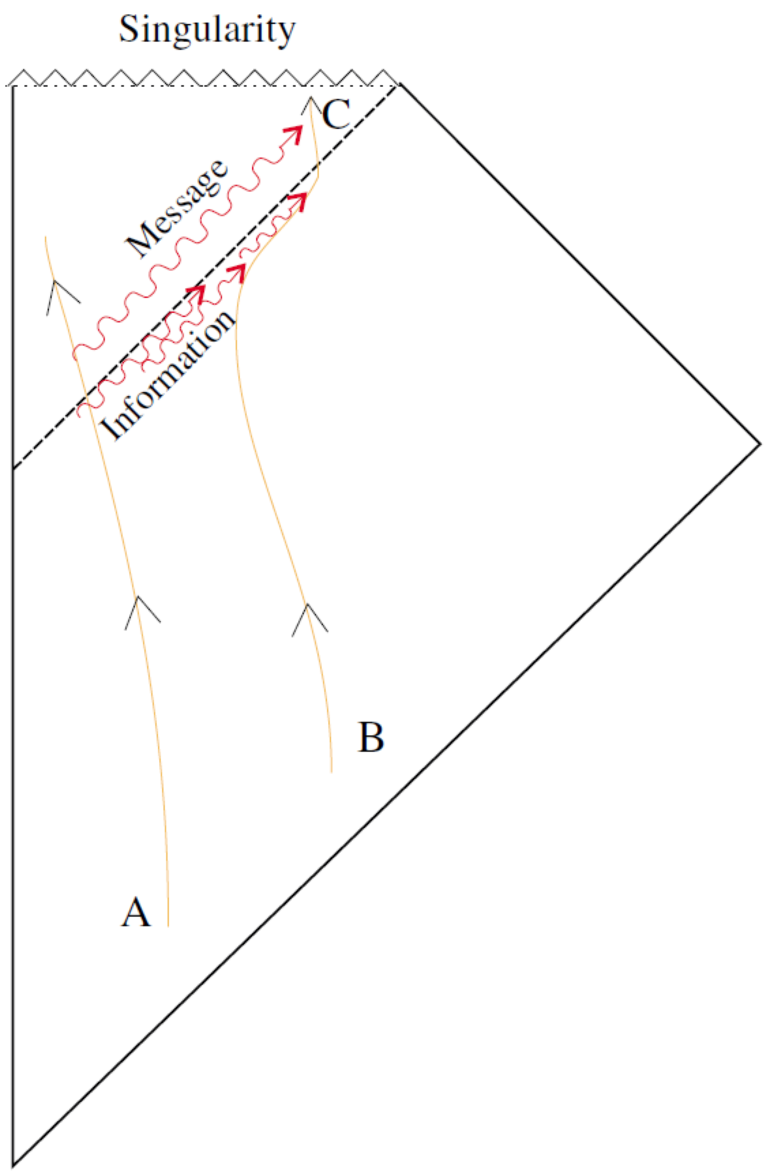
# Black Hole Complementarity

1. There exists a unitary S-matrix which describes the evolution from in-falling matter to outgoing Hawking-like radiation. (Unitarity)
2. Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations. (QFT in curved space-time)
3. A freely falling observer experiences nothing out of the ordinary when crossing the horizon. (EP)

# Black Hole Complementarity

- One strategy was proposed by Susskind, Thorlacuis & Uglam in 1993

# Black Hole Complementarity




From Susskind 2004

# Black Hole Complementarity

- Black hole evaporates in a time of $\tau = M^3$ so page time would be the of the same order:
$$t_{page} = M^3$$
- Observer B hovers above horizon at a distance at least of the order $l_p$ i.e. above the stretched horizon. Thus we adapt near horizon coordinates:
$$ds^2 = -(\kappa x)^2 dt^2 + dx^2 = -\rho^2 d\omega^2 + d\rho^2$$
- Thus:
$$\omega = \frac{t}{4M} \Rightarrow \omega^* \geq M^2, \qquad \rho \geq l_p$$
- Introducing time cone coordinates $x^{\pm} = \rho e^{\pm\omega}$ we would have:
$$x_*^+ x_*^- \geq l_p^2, \qquad x_*^+ \geq l_p \exp(\omega^*)$$

# Black Hole Complementarity

- The singularity is given by:

$$x^+x^- = M^2$$

- If A is to send the signal before B hits the singularity one has:

$$x^- \leq Me^{-\omega^*}$$

- Then from time-energy uncertainty A's energy would be:

$$E > \frac{1}{M}\, e^{M^2}$$

- A cannot fit into the horizon!

# Black Hole Complementarity

- Another Strategy was due to Haydan & Preskill in 2007.
- This strategy demonstrates that one just barely fails to observe Xeroxing. The energy required for A would be:

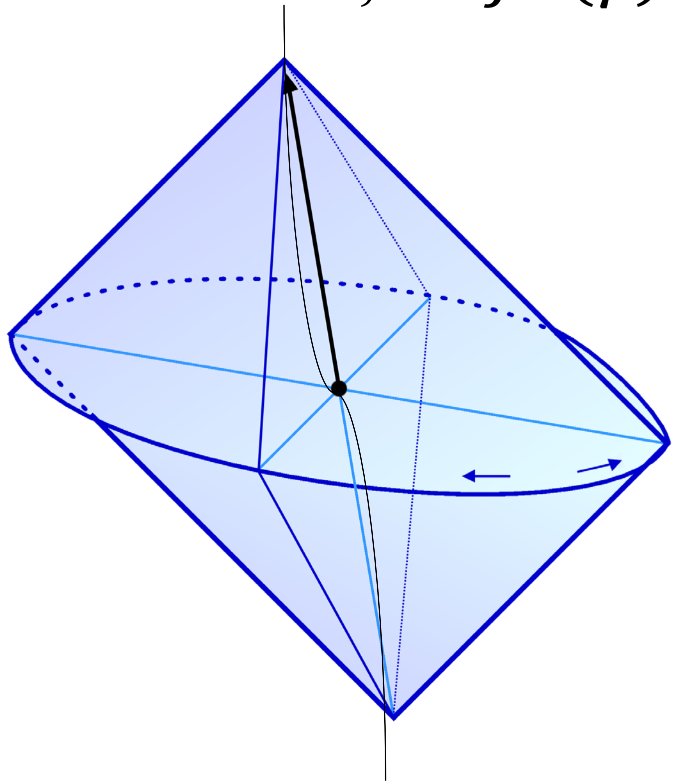
$$E = O(MlogM)$$

# Black Hole Complementarity

- It seems the following statements cannot be consistent:
  1. Unitarity
  2. EP
  3. Omniscience: Existence of a consistent description of the entire space-time, even if there are horizons, i.e. even if no-one can observe the entire space-time.

# Black Hole Complementarity

- Complementarity: A fundamental description of Nature need only describe experiments that are consistent with causality. The regions that can be probed are causal diamonds, i.e. $J^{+}(\gamma) \cap J^{-}(\gamma)$ for each world line $\gamma$.



# Black Hole Complementarity

- Is BHC too philosophical?!
- Susskind and Thorlacius basically say that if nobody can see the quantum state on a whole slice, why should it exist in the first place?
- This type of argument does have at least one very successful historical analogue; the Heisenberg Uncertainty principle.

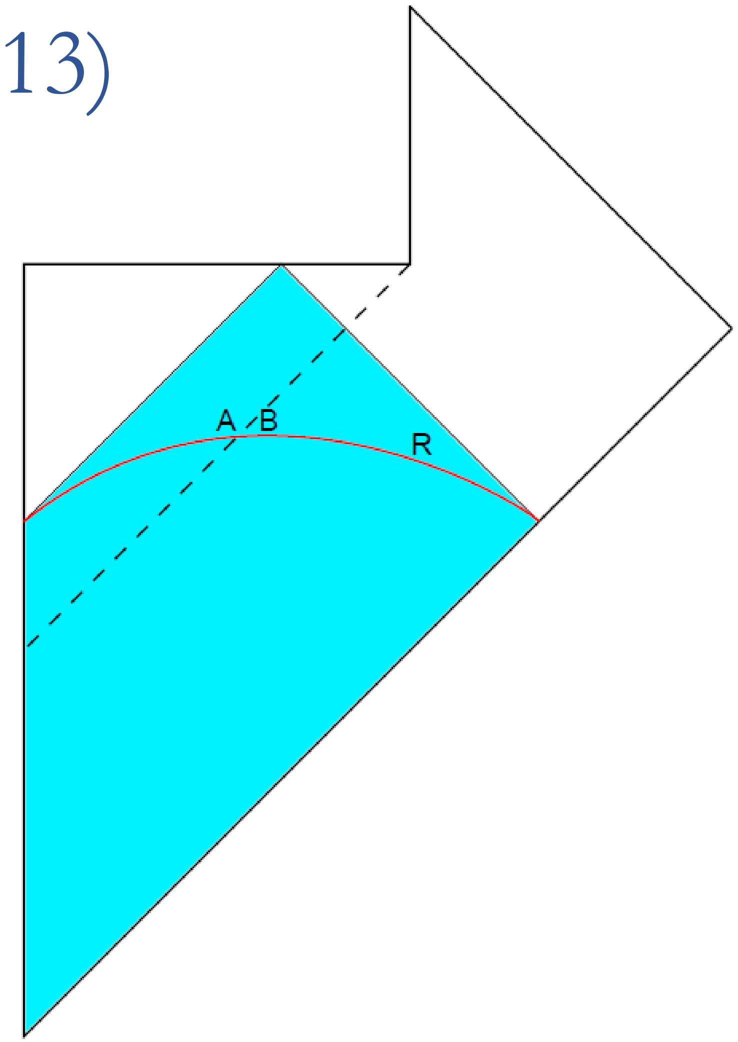
# Firewalls (AMPS 2013)

- AMPS claim that these 3 statements cannot all be true:
  1. Purity of Hawking radiation.
  2. Outside the stretched horizon, low energy effective field theories are valid.
  3. EP
- The goal of the AMPS argument is to put all of the "moving parts" of the black hole information problem into the past lightcone of a single observer, preventing any use of complementarity to avoid an observable violation of effective field theory or quantum mechanics.

# Firewalls (AMPS 2013)

- We review AMPS argument in 2 different languages first we use the concept of entanglement, which is the more intuitive way.
- We divide the hawking radiation in three subsystems, the early radiation R, the late radiation B and the interior partners to the late radiation A.
- "Late" means after the time $t_{page}$ when most of the mass of the black hole is radiated away.
- For radiation to become pure it must be true that R and B are almost maximally entangled. (Page 1993)

# Firewalls (AMPS 2013)

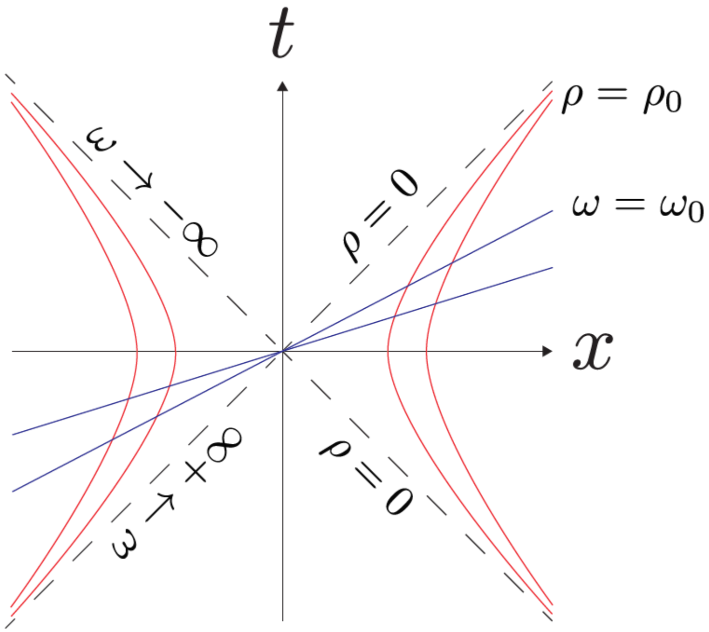



From Harlow 2015

# Firewalls (AMPS 2013)

- From EP, one expects the in-falling observer to see Minkowski vacuum at the horizon.
- Using the Rindler coordinates near the horizon, one can decompose the Hilbert space:

$$H = H_R \otimes H_L$$



# Firewalls (AMPS 2013)

- Thus for eigen-states of the field operator and Hamiltonian one has:

$$|\phi_M\rangle = |\phi_R\rangle \otimes [\Theta|\phi_L\rangle], \qquad |n\rangle = |n_R\rangle \otimes [\Theta|n_L\rangle]$$

- Then we can calculate the following amplitude:

$$\langle\phi_M|0_M(t=0)\rangle \propto \lim_{T\to\infty}\langle\phi_M|e^{-HT}|\phi(t_E=-T)\rangle \propto \int_{\phi(t_E\to-\infty)=0}^{\phi(t_E=0)=\phi_M} D\phi e^{-I_E}$$

$$\Rightarrow \langle\phi_M|0_M(t=0)\rangle \propto \int_{\phi(\theta\to-\pi)=\phi_L}^{\phi(\theta=0)=\phi_R} D\phi e^{-I_E} \propto \langle\phi_R\left|e^{-\pi H^R}\right|\phi_L\rangle$$

- After some calculation:

$$\langle\phi_M|0_M(t=0)\rangle \propto \langle\phi_R| \otimes \left(\langle\phi_L|\Theta^\dagger\right)[\textstyle\sum_n e^{-\pi E_n}\ |n_R\rangle \otimes (\Theta|n_L\rangle)]$$

# Firewalls (AMPS 2013)

- Thus one defines Thermo-field double state:

$$|0_M(t=0)\rangle = |TFD\rangle = \sum_n \frac{e^{-\frac{\beta}{2}E_n}}{\sqrt{Z(\beta)}} |n_R\rangle \otimes [\Theta|n_L\rangle]$$

- Consequently, the vacuum of Minkowski space is an entangled state of the Hamiltonian eigen-states $\{|n_R\rangle, |n_L\rangle\}$.

# Firewalls (AMPS 2013)

- Thus for in-falling observer to fall in without drama, one realizes that B and A must be nearly maximally entangled as well.
- Maximal entanglement of A with B and B with R, violates the strong subadditivity of the entropy or Monogamy of the entropy.

# Firewalls (AMPS 2013)

- To express AMPS thought experiment in another language which is more precise, we remember the postulates of complementarity:

1. There exists a unitary S-matrix which describes the evolution from in-falling matter to outgoing Hawking-like radiation. (Unitarity)
2. Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations. (QFT in curved space-time)
3. A freely falling observer experiences nothing out of the ordinary when crossing the horizon. (EP)

# Firewalls (AMPS 2013)

- From postulate 1, we know that the full state of the BH+radiation is a pure state:

$$|\Psi\rangle = \sum_i |\psi_i\rangle_E \otimes |i\rangle_L$$

- When BH has emitted at least half of its initial Bekenstein-Hawking entropy, the early radiation subspace would be much larger than the late subspace. As a result these two subspaces would be highly entangled due to Page's theorem, confirming our first statement of AMPS argument.

# Firewalls (AMPS 2013)

- If we consider an outgoing mode of the radiation calling it $i$, then by postulate 2, outside the horizon we have the corresponding unique operator $b_i$ for that mode as long as we are outside the stretched horizon.
- One can project states to eigen-spaces of the number operator $N_i = b_i^\dagger b_i$, and it is natural that $|\Psi\rangle$ would be an eigen-state of $N_i$. (We expect to have a certain number of photons with the frequency $\omega_i$ in the radiation.)

# Firewalls (AMPS 2013)

- Since we specified $b_i$ on some Cauchy surface in the future, using postulate 2 we can calculate the evolution of modes to distances outside the stretched horizon.
- If we look at the in-falling observer, EP tells us that she should see nothing special, i.e. Minkowski vacuum. Modes would have corresponding operators, $a_\omega$. then we can write:

$$b_i = \int_0^\infty d\omega \left( B(\omega) a_\omega + C(\omega) a_\omega^\dagger \right)$$

- In-falling observer sees vacuum: $a_\omega |\Psi\rangle = 0$. This equivalent to the entanglement of the black hole and the late radiation

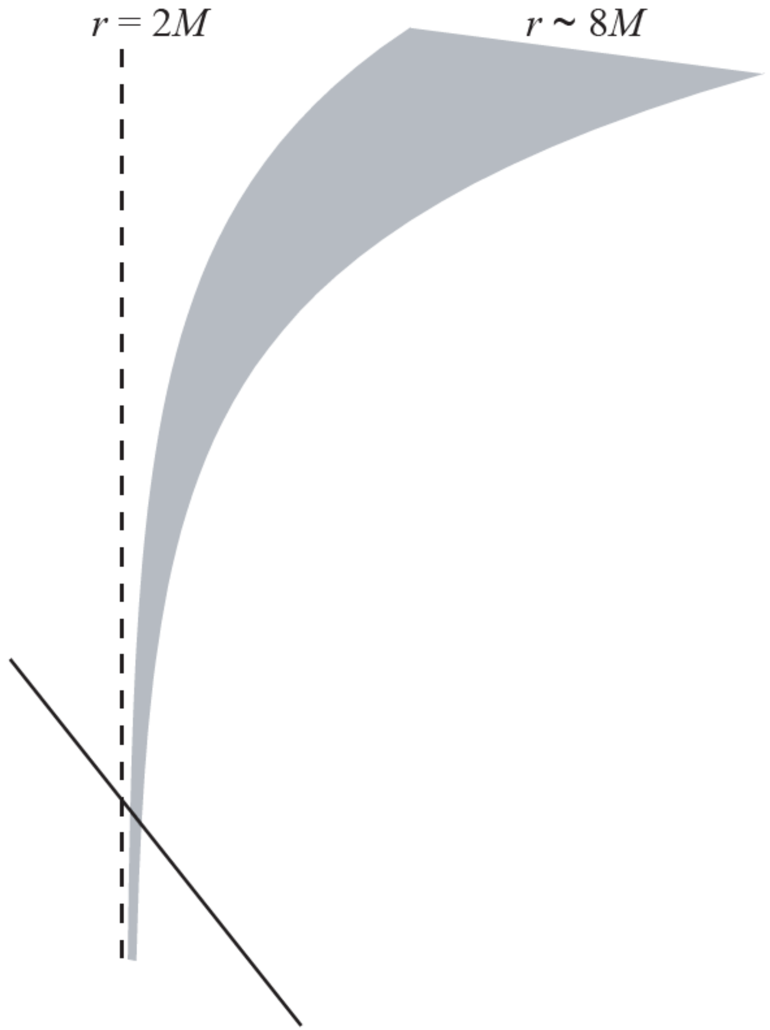
# Side-Note: Entanglement of Late Radiation and It's Partner?!

- Heuristic argument: Hawking pairs!
- Using TFD state it is shown that the full Minkowski vacuum can be reinterpreted as the thermofield double in two copies of Rindler space.
- If we accept that EP means Minkowski vacuum, then we have an entangled TFD state.
- So the argument of monogamy is relevant because one can purify the purification of the late radiation to be the TFD, but why one wedge is the interior ?

# Firewalls (AMPS 2013)

- If all axioms of BHC are to hold:
- From postulate 1 at far photons are eigen-states of $N_i = b_i^\dagger b_i$.
- Closer to the horizon at shorter wavelengths due to blueshift, field is in the vacuum state from postulate 3.
- From postulate 2, one can follow the evolution of modes close to the stretched horizon so there field should be an eigen-state of $N_i$.
- But if $a_\omega|\Psi\rangle = 0$, from the Bogoliubov transformation on the last page one has: $\langle\Psi|N_i|\Psi\rangle = 0$ which is a contradiction.

# Firewalls (AMPS 2013)




From AMPS 2013

# Firewalls (AMPS 2013)

- So we must give up one of these:

1. Unitarity (Entanglement of the early and late radiation)
2. Validity of QFT outside stretched horizon (Evolution of the modes from the near to the far)
3. Equivalence Principle (Entanglement of the late radiation and it's interior partner)

- AMPS argue that the most conservative alternative is to question the validity of the Equivalence Principle. This implies that the state of near horizon radiation is significantly different from that of the vacuum, leading to the conclusion that the observer must see high energy quanta.
- This high energy quanta is interpreted as hitting a "Firewall" !

# Interior Region

- When we discussed the stretched horizons and then in our further arguments involving BHC, it seemed like we essentially ignored the interior of the black hole.
- If black hole complementarity is consistent and correct, shouldn't we be able to find a real theory of the interior that realizes it?

# Interior Region

- Susskind claims that BHC states that the information in the interior of a black hole is meaningful, but it is redundant with information in the exterior of the black hole. At early time, before there's been much evaporation, the redundancy is between the interior and the stretched horizon. Later, after a great deal of evaporation, the redundancy is between the interior and the Hawking radiation. The interior is meaningful, but it, and the Hawking radiation, should not be counted as independent.
- I will explain why the above statement can be reasonable further on.

# Proximity Postulate

- Proximity Postulate states that the interior of a black hole is constructed from the degrees of freedom near the horizon.
- The statement in the last slide is in conflict with the proximity postulate, because radiation degrees of freedom are far from the black hole.

# Break down of the Proximity Postulate.

- In the following Slides we will show that:
- AMPS implicit assumption is the proximity postulate: the interior of a black hole must be constructed from degrees of freedom that are physically near the black hole.
- AMPS argue that a violation of the proximity postulate would lead to a contradiction and that the only way to protect against the contradiction is for a Firewall to form at the Page time.
- Harlow and Hayden argue against this with a conjecture based on computational complexity.

# Scrambling

- If one considers a highly non-typical state from a $2^N$ dimensional Hilbert space,

$$|\psi_0\rangle = |000 \ldots 0\rangle.$$

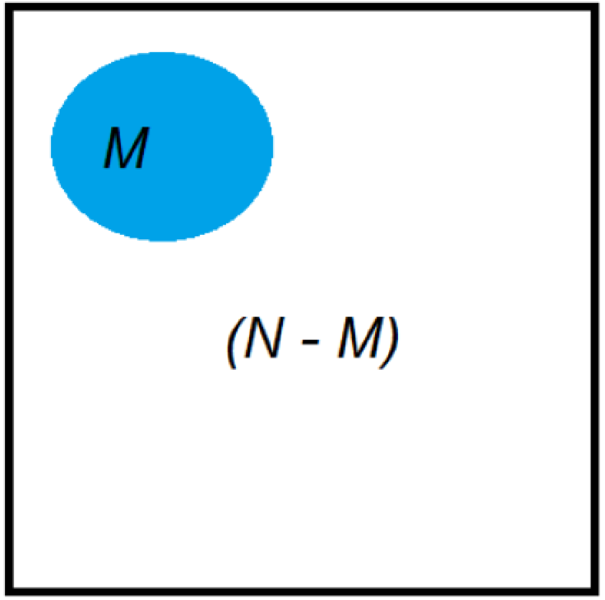
- Then one can scramble the state above using a random Haar unitary matrix. Then the scrambled state is defined by,

$$|\psi(U)\rangle = U|\psi_0\rangle.$$

- By Page's theorem With overwhelming probability $|\psi(U)\rangle$ has the scrambled property; namely, any small sub-system has essentially no information (i.e. $S_{small} \approx \dim(small)\, log2$).

# Scrambling

- A small subsystem means any subset of qubits fewer than half the total number. If $M < N/2$; then a system of $M$ qubits is small.



# Scrambling

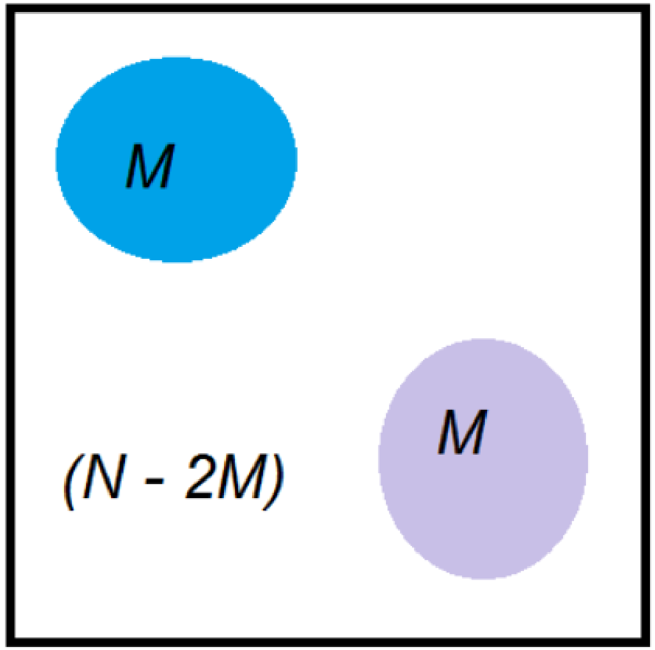
- A scrambled state can be written in the form:

$$|\psi(U)\rangle = \sum_i |i\rangle_s |\phi_i\rangle_b,$$

- Where $|i\rangle_s$ are basis vectors of small subsystem, and $|\phi_i\rangle_b$ are states in the big system.
- Because the density matrix of the small subsystem is maximal, then $|\phi_i\rangle_b$ have to be orthonormal.

# Scrambling

- If we consider the following system:



# Scrambling

- Any vector of the form $|\psi(U)\rangle = \sum_i |i\rangle_s |\phi_i\rangle_b$, is close to a vector that can be expressed by a two step process.
- First we define a state in which the two small subsystems are maximally entangled,

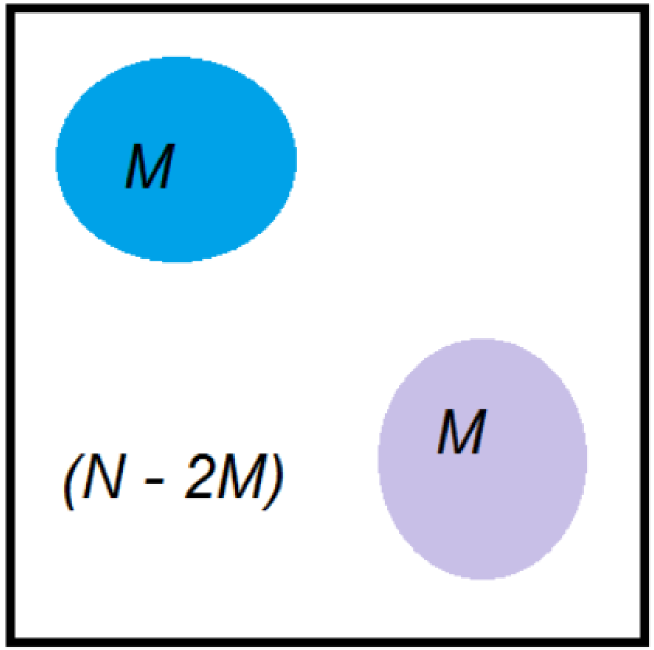
$$|\phi\rangle = \sum_i |i\rangle_s |i\rangle_{s'} |000\ldots\rangle\,.$$

- Then we scramble the second $N - M$ qubits to arrive at $|\psi(U)\rangle$,

$$|\psi(U)\rangle = V|\phi\rangle$$

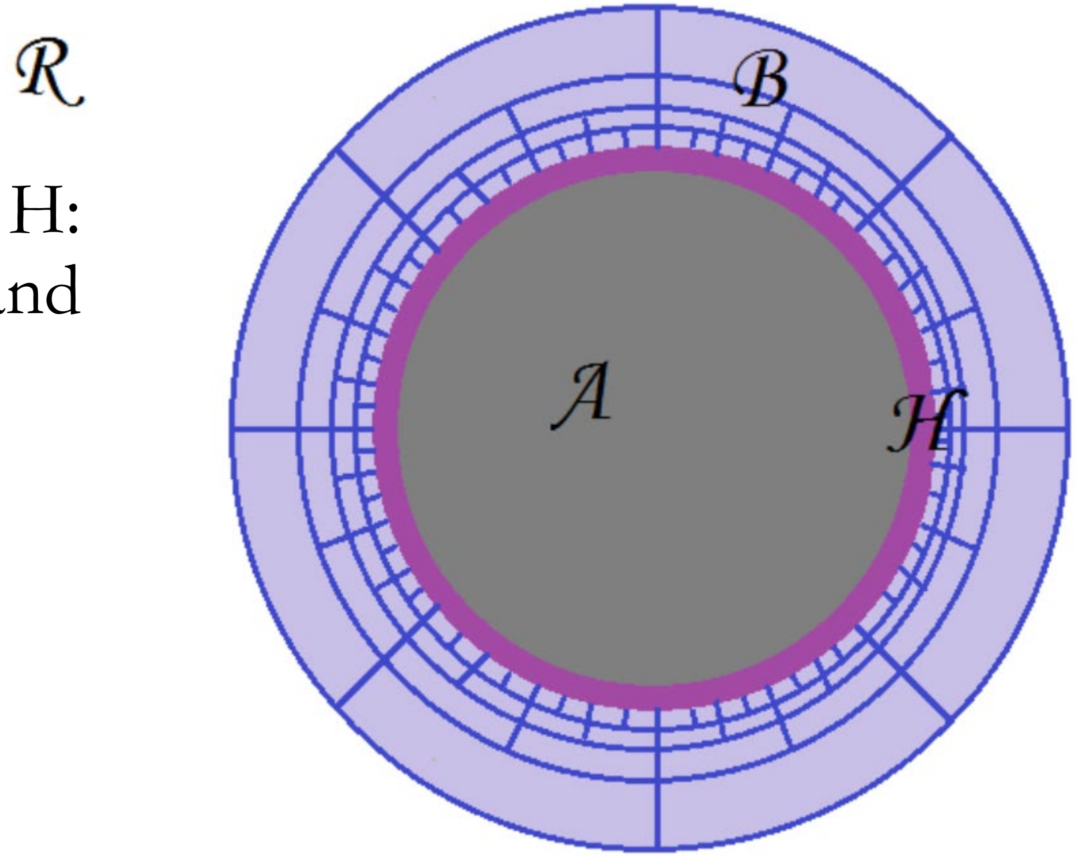
# Scrambling

- The point to make is that in an N qubit system, almost all states are of the form $|\psi(U)\rangle$, where an small M qubit subsystem is entangled with a M qubit subsystem of the big N-M subspace.



# Degrees of Freedom of a Black Hole

- We assume that the degrees of freedom of a black hole are fully specified with regions R,B and H: the stretched horizon, atmosphere and the radiation.
- Thus the full state of black hole is expressed with the above regions.



# A↔H

- Due to no drama at the Horizon, as was previously shown, modes $A_i$ must be maximally entangled with modes $B_i$.
- For relatively young black holes, with little evaporation but enough scrambling one expects $BH$ to be in a pure scrambled state.
- Due to properties of scrambled states, $B$ is maximally entangled with $H$. And also each mode $B_i$ is maximally entangled with a subspace $H_{B_i}$

# A↔H

1. $B_i \leftrightarrow A_i$
2. $B_i \leftrightarrow H_{B_i}$
3. Monogamy

$$\left.\begin{array}{l} 1.\ B_i \leftrightarrow A_i \\ 2.\ B_i \leftrightarrow H_{B_i} \\ 3.\ \text{Monogamy} \end{array}\right\} \quad A_i \leftrightarrow H_{B_i}$$

# Distillation of Entanglement

- As a measure of entanglement for states one can consider the ditilled entanglement, $D$. This is roughly the number of bell states that can encompass the information in n qubits.
- For a mixed state like the region BH this quantity is bounded,

$$D \leq \mu, \qquad \mu = \frac{1}{2}(\mathrm{S_B} + \mathrm{S_H} - \mathrm{S_{BH}}) = \frac{1}{2}(\mathrm{S_B} + \mathrm{S_H} - \mathrm{S_R}).$$

- There are two cases where $D$ is easy to compute, $\mu = 0 \Rightarrow D = 0$ & $\mu = Maximal \Rightarrow D \approx \mu$.

# Loss of Entanglement of H & B

- As long as H is greater than half the system HBR, by ideas of scrambling:

$$S_{\mathrm{B}} = \mathrm{N}_{\mathrm{B}}, \qquad \mathrm{S}_{\mathrm{R}} = \mathrm{N}_{\mathrm{R}}, \qquad \mathrm{S}_{\mathrm{H}} = \mathrm{S}_{\mathrm{B}} + \mathrm{S}_{\mathrm{R}}$$

$$\Rightarrow \mu_{BH} = N_B \Rightarrow D_{BH} = N_B$$

- This behavior holds so long as H is greater than half of the system. The time which H is half of the system, denoted by $t_c$ is called the cusp time. It is less than $t_{page}$ which is the time that R is the half of the system. At $t_c$, $N_H = N_R + N_B$, so we define, $N_c = N_H - N_B$.

# Loss of Entanglement of H & B

- After the time $t_c$, $\mu_{BH}$ decreases linearly with time. It vanishes at $t_{page}$ and stays equal to zero until the black hole evaporates.
- To see why this is the case we note that for $t_c < t < t_{page}$, All subsystems are less than half of the total system. Then by notions of scrambling, $S_B = N_B, S_H = N_H, \& \ S_R = N_R$. So,

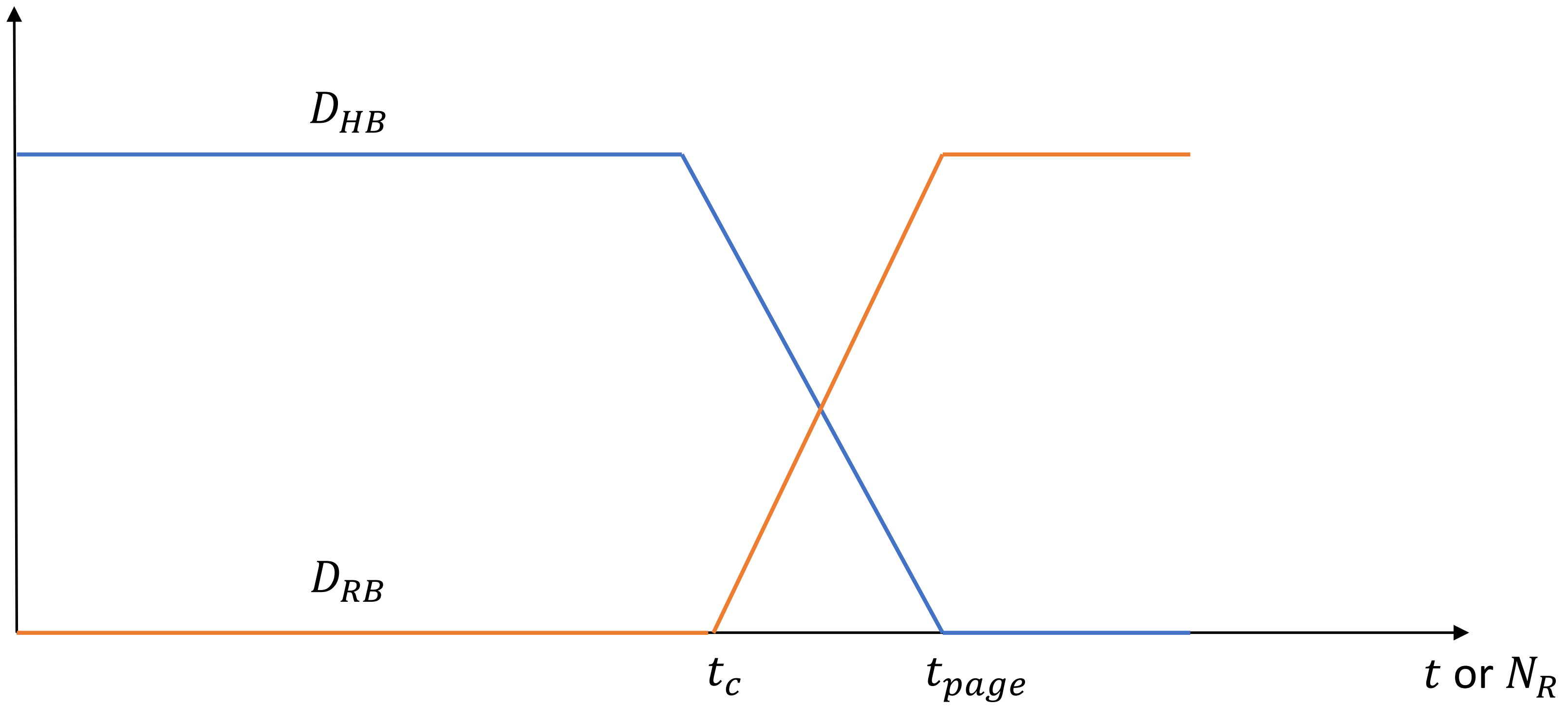
$$\mu_{BH} = \frac{1}{2}(N_B + N_H - N_R) = \mathrm{N_B} - \frac{\mathrm{N_R} - \mathrm{N_c}}{2}.$$

# Loss of Entanglement of H & B

- From the fact that $\mu$ bounds $D$ we see that the distillable entanglement between H and B also decreases to zero at the Page time.

# Evaporation

- The whole argument in the previous slides in summarized in the following figure:



# $A \leftrightarrow R$ ?

- After the Page time the degree of freedom that is maximally entangled with $B_i$ lives in $R$ and can be called $R_{B_i}$. By the same argument as before we can say $A_i \leftrightarrow R_{B_i}$ .
- One can argue that the statement above just states that region $A$, cannot be constructed from the neat horizon degrees of freedom(breakdown of the proximity postulate). And it is not an inconsistency with the original BHC postulates.
- AMPS argues against the claim above.

# A↔R ?

- AMPS try to rule out the claim we made in the last slide by a thought experiment.
- Alice + Computer
- Alice collects radiation after $t_{page}$ and extracts an $R_{B_i}$ corresponding to an $A_i$ by hypothesis.
- If $A_i$ is a degree well after $t_{page}$, then if Alice's computer is strong enough she knows the information in $A_i$ before it has happened.
- Alice then jumps in with $R_{B_i}$, just in time to see $A_i$ and $B_i$ and claims mockingly that monogamy is destroyed once more!

# A↔R ?

- With the last argument, AMPS claim that the problem is not solved by accepting the loss of proximity postulate.
- They claim that the only way to protect against the contradiction is for a Firewall to form at the Page time.

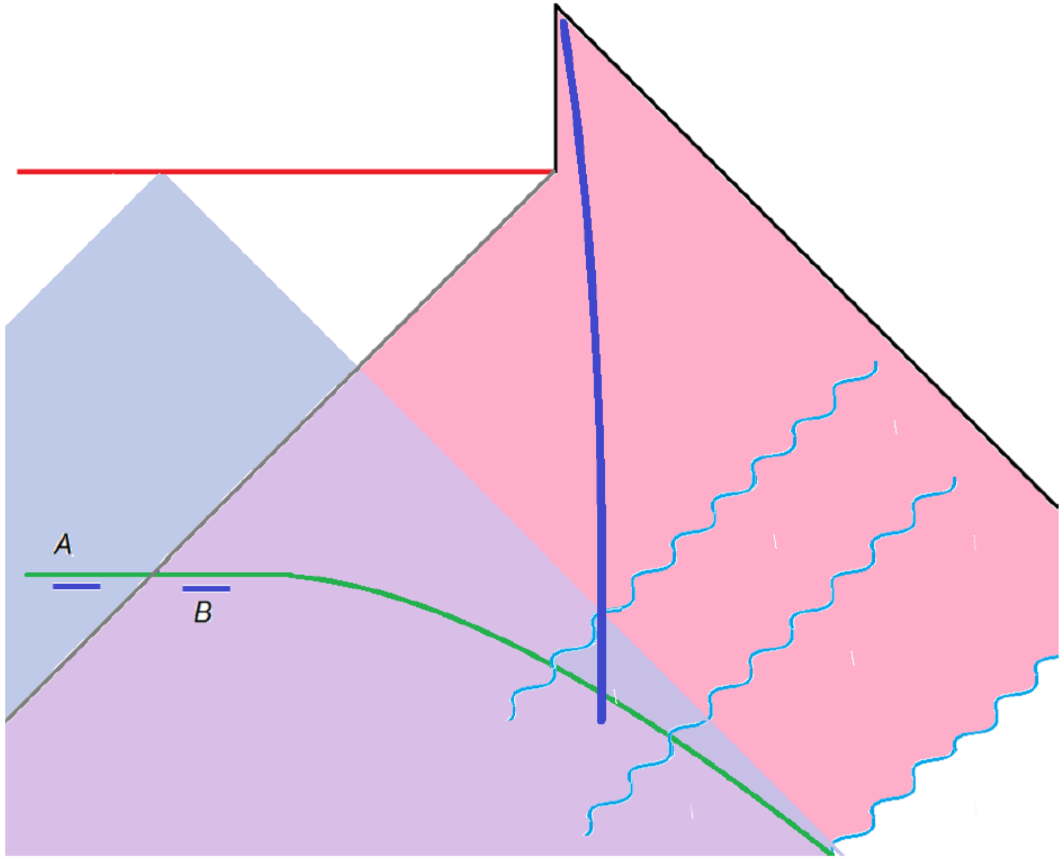
# Harlow-Hayden Conjecture (2013)

- If we consider the black hole to be a system of N qubits:
- *HH Conjecture: The minimum time that it takes to distill qubit $R_{B_i}$, exponentially grows with N.*
- For an ordinary Schwarzschild black hole the total time before it evaporates is of only of order $N^3$: Thus the truth of the HH conjecture would undermine the thought experiment designed to prove the existence of Firewalls.

# Harlow-Hayden Conjecture (2013)

- Thus if HH conjecture is valid, only Proximity is violated. And thus interior can in principle be redundant in the radiation, opening up a lot of new possibilities.

# Harlow-Hayden Conjecture (2013)



# Summary

- Black holes form and evaporate, radiating thermal radiation.
- This process leads to the loss of information and violation of the Unitarity.
- Unitarity + EP → Xeroxing paradox
- Unitarity + EP + Ominscience (BHC) → apparently resolves the paradox.
- AMPS paradox → BHC is not enough.
- With the realization of the proximity postulate one can claim that AMPS argument only violates that.
- AMPS uses distilling of entanglement to argue against the claim above.
- HH conjectures that distilling before evaporation is impossible in principle.

# Questions that Remain for Me

1. Page's theorem for $\bigoplus_{\alpha} (H_{A_\alpha} \otimes H_{B_\alpha})$.
2. The exact definition and understanding of the stretched horizon.
3. Entanglement of radiation and it's interior partners? Because my argument of thermofield doubles seems to be incomplete (thermofield subsystems are highly entangled, but not maximally) and the hawking pair argument is heuristic.
4. The exact understanding of the degrees of freedom RHB and how does BH describe the black hole and at the same time interior A is separated.
5. Why the model evaporation used by Susskind is correct physically?
6. A review of distillation of entanglement.

# Refereneces


- S.W. Hawking, Particle creation by black holes, Comm. Math. Phys. 43 (1975) 199-220.
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- Don N. Page. Information in Black Hole Radiation, hepth/ 9306083, Phys. Rev. Lett. 71 (1993) 3743–3746.
- J. Lindesay, L. Susskind, An Introduction to Black Holes, Information and the String Theory Revolution: The Holographic Universe, World Scientific (2004).
- Raphael Bousso’s Talk, Black Holes: Complementarity vs Firewalls
- A. Almheiri, D. Marolf, J. Polchinski and J. Sully, \Black Holes: Complementarity or Firewalls?," arXiv:1207.3123 [hep-th].
- Daniel Harlow, Jerusalem Lectures on Black Holes and Quantum Information.
- L. Susskind, Black Hole Complementarity and the Harlow-Hayden Conjecture.

# Thank you