به نام خدا

پاسخ نامه سری اول تمرینات مکانیک کوانتوم ۲.

سوال ۱ :

الف)

$$\psi(\theta,\varphi) = \frac{1}{\sqrt{5}} Y_{1,-1}(\theta,\varphi) + \frac{\sqrt{3}}{\sqrt{5}} Y_{1,0}(\theta,\varphi) + \frac{1}{\sqrt{5}} Y_{1,1}(\theta,\varphi)$$

$$\longrightarrow |\psi\rangle = \frac{1}{\sqrt{5}} |1,-1\rangle + \sqrt{\frac{3}{5}} |1,0\rangle + \frac{1}{\sqrt{5}} |1,1\rangle$$

$$L_+ |l,m\rangle = \hbar\sqrt{l(l+1)-m(m+1)}\ |l,m+1\rangle$$

$$\langle l,m'|L_+|l,m\rangle = \hbar\sqrt{l(l+1)-m(m+1)}\ \langle l,m'|l,m+1\rangle$$

$$\delta_{m',m+1} \longrightarrow m'=m+1$$

$$\langle\psi|L_+|\psi\rangle = \frac{\sqrt{3}}{\sqrt{5}}\frac{1}{\sqrt{5}} \langle 1,0|L_+|1,-1\rangle + \frac{\sqrt{3}}{\sqrt{5}\sqrt{5}} \langle 1,1|L_+|1,0\rangle$$

$$= \frac{\sqrt{3}}{5}\hbar \underbrace{\sqrt{1(1+1)-(-1)(-1+1)}}_{\sqrt{2}} + \frac{\sqrt{3}}{5}\hbar \underbrace{\sqrt{1(1+1)-0(0+1)}}_{\sqrt{2}} = \frac{2\sqrt{6}}{5}\hbar$$

ب) مقادیر ویژه $\hat{L}_z$:

$$m \in \{0, \pm 1\} \longrightarrow m\hbar : \{0, \pm\hbar\}$$

$$P(m=0) = |\langle 1,0|\psi\rangle|^2 = \frac{3}{5}$$

$$P(m=+1) = |\langle 1,1|\psi\rangle|^2 = \frac{1}{5}$$

$$P(m=-1) = |\langle 1,-1|\psi\rangle|^2 = \frac{1}{5}$$

ج) اگر $l_z=-\hbar$ اندازه گیری شود حالت سیستم به $|\psi\rangle = |1,-1\rangle$ تقلیل می‌یابد.

$$\Delta\hat{A} = \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2}$$

از تقارن مسئله $\longrightarrow \langle L_x\rangle = \langle L_y\rangle = 0$

$$\langle L_x^2\rangle = \langle L_y^2\rangle$$

$$L^2 = L_x^2 + L_y^2 + L_z^2$$

$$L_x^2 = L_y^2 = \frac{1}{2}(L^2 - L_z^2)$$

$$\langle L_x^2\rangle = \langle L_y^2\rangle = \left\langle \tfrac{1}{2}(L^2 - L_z^2)\right\rangle = \langle \psi | \tfrac{1}{2}(L^2 - L_z^2)|\psi\rangle$$

$$= \frac{1}{2}\langle 1,-1| L^2 - L_z^2 |1,-1\rangle = \frac{\hbar^2}{2}\left(l(l+1) - m^2\right)\Big|_{\substack{l=1\\ m=-1}} = \frac{\hbar^2}{2}$$

$$\longrightarrow \Delta L_x = \Delta L_y = \sqrt{\langle L_x^2\rangle} = \sqrt{\frac{\hbar^2}{2}} = \frac{\hbar}{\sqrt{2}}$$

$$\longrightarrow \Delta L_x \, \Delta L_y = \frac{\hbar^2}{2}$$

سوال ۲ :

الف)

$$H = \varepsilon\, \vec{\sigma}\cdot\hat{n}$$

$$\hat{n} = (\sin\theta\cos\varphi\,,\ \sin\theta\sin\varphi\,,\ \cos\theta)$$

$$\sigma_x = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} & -i \\ i & \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\vec{\sigma}\cdot\hat{n} = \begin{pmatrix} \cos\theta & \sin\theta(\cos\varphi - i\sin\varphi) \\ \sin\theta(\cos\varphi + i\sin\varphi) & -\cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}$$

ویژه مقدار : $\det(\vec{\sigma}\cdot\hat{n} - \lambda I) = 0 \longrightarrow \det\begin{pmatrix} \cos\theta - \lambda & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta - \lambda \end{pmatrix} = 0$

$$\longrightarrow -(\cos\theta + \lambda)(\cos\theta - \lambda) - \sin^2\theta = 0$$

$$-\cos^2\theta + \lambda^2 - \sin^2\theta = 0 \longrightarrow \lambda^2 = 1 \longrightarrow \lambda = \pm 1$$

ویژه مقادیر هامیلتونی : $E = \varepsilon\lambda \longrightarrow E = \pm\varepsilon$

ویژه بردارها

$$\lambda = +1 \longrightarrow |n+\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \qquad \vec{\sigma}\cdot\hat{n}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \longrightarrow \begin{cases} a = a\cos\theta + b\sin\theta\, e^{-i\varphi} \\ b = \sin\theta\, e^{i\varphi} a - \cos\theta\, b \end{cases}$$

$$\longrightarrow a(1-\cos\theta) = b\sin\theta\, e^{-i\varphi} \longrightarrow a\left(\cancel{2}\sin^{\cancel{2}}\theta/2\right) = b\left(\cancel{2}\cancel{\sin\theta/2}\cos\theta/2\right)e^{-i\varphi}$$

$$\longrightarrow \begin{cases} a = b\cot\theta/2\; e^{-i\varphi} \\ a^2+b^2 = 1 \end{cases}$$

$$b = \sin\theta/2 \longrightarrow a = \cos\theta/2\; e^{-i\varphi} \longrightarrow \begin{pmatrix} \cos\theta/2\; e^{-i\varphi} \\ \sin\theta/2 \end{pmatrix}$$

بردار بالا ضرب در هر فازی ویژه بردار $\hat{n}\cdot\vec{\sigma}$ با ویژه مقدار $\lambda=+1$ است که برای نوشتن بردار می‌توان آن را در $e^{i\varphi/2}$ ضرب کرد:

$\lambda=+1$

$$|n+\rangle = \begin{pmatrix} \cos\theta/2\; e^{-i\varphi/2} \\ \sin\theta/2\; e^{i\varphi/2} \end{pmatrix}$$

به طور مشابه

$\lambda=-1$

$$|n-\rangle = \begin{pmatrix} -\sin\theta/2\; e^{-i\varphi/2} \\ \cos\theta/2\; e^{i\varphi/2} \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\theta/2\; e^{-i\varphi} & -\sin\theta/2\; e^{-i\varphi} \\ \vdots & \\ \sin\theta/2\; e^{i\varphi} & \cos\theta/2\; e^{i\varphi/2} \end{pmatrix}$$

ب)

$$e^{-i\alpha\hat{n}\cdot\vec{\sigma}} = \cos\alpha\, I - i\sin\alpha\,(\hat{n}\cdot\vec{\sigma})$$

$$\hat{n}\cdot\vec{\sigma} = (+1)\,|n+\rangle\langle n+| + (-1)\,|n-\rangle\langle n-| = |n+\rangle\langle n+| - |n-\rangle\langle n-| = \begin{pmatrix} +1 & \\ & -1 \end{pmatrix}$$

$$e^{-i\alpha\hat{n}\cdot\vec{\sigma}} = e^{-i\alpha}\,|n+\rangle\langle n+| + e^{i\alpha}\,|n-\rangle\langle n-| = \begin{pmatrix} e^{-i\alpha} & \\ & e^{i\alpha} \end{pmatrix}$$

نمایش ماتریسی در پایه $\{|n\pm\rangle\}$

$$= \begin{pmatrix} \cos\alpha - i\sin\alpha & 0 \\ 0 & \cos\alpha + i\sin\alpha \end{pmatrix} = \cos\alpha \underbrace{\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}}_{I} + (-i\sin\alpha)\underbrace{\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}}_{\hat{n}\cdot\vec{\sigma}}$$

$$e^{-i\alpha\,\hat{n}\cdot\vec{\sigma}} = \cos\alpha\, I - i\sin\alpha\,(\hat{n}\cdot\vec{\sigma})$$

$$U(t) = e^{-iHt/\hbar} = e^{-i\left(\frac{\varepsilon}{\hbar}t\right)\hat{n}\cdot\vec{\sigma}} = \cos\left(\frac{\varepsilon t}{\hbar}\right) I - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\hat{n}\cdot\vec{\sigma}$$

$$|\psi(0)\rangle = \begin{pmatrix}1\\0\end{pmatrix} = |z+\rangle$$ 12.

$$P\left(l_z = -\frac{\hbar}{2}\right) = |\langle z-|z+\rangle|^2 = 0$$

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = \left[\cos\left(\frac{\varepsilon t}{\hbar}\right) I - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\underbrace{\hat{n}\cdot\vec{\sigma}}_{\cos\theta\,\sigma_z + \sin\theta\cos\varphi\,\sigma_x + \sin\theta\sin\varphi\,\sigma_y}\right]|z+\rangle$$

$$= \left[\cos\left(\frac{\varepsilon t}{\hbar}\right) - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\cos\theta\right]|z+\rangle - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\sin\theta\left(\cos\varphi\,\frac{\sigma_+ + \sigma_-}{2} + \sin\varphi\,\frac{\sigma_+ - \sigma_-}{2i}\right)|z+\rangle$$

$$= \left(\cos\left(\frac{\varepsilon t}{\hbar}\right) - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\cos\theta\right)|z+\rangle - i\sin\left(\frac{\varepsilon t}{\hbar}\right)\sin\theta\left(\frac{\cos\varphi - i\sin\varphi}{2}\right)\underbrace{\cancel{\sigma_+|z+\rangle}}_{0}$$

$$- i\sin\left(\frac{\varepsilon t}{\hbar}\right)\sin\theta\left(\frac{\cos\varphi + i\sin\varphi}{2}\right)\underbrace{\sigma_-|z+\rangle}_{2|z-\rangle}$$

$$\longrightarrow \quad P\left(l_z = -\frac{\hbar}{2}\right) = |\langle z-|\psi(t)\rangle|^2 = \sin^2\theta\,\sin^2\left(\frac{\varepsilon t}{\hbar}\right)$$

سوال ۳ :

الف)

$$H = \frac{L_x^2+L_y^2}{2I_1} + \frac{L_z^2}{2I_2} = \frac{L^2-L_z^2}{2I_1} + \frac{L_z^2}{2I_2}$$

ویژه مقادیر : $\dfrac{\hbar^2 l(l+1)}{2I_1} + \hbar^2 m^2 \left(\dfrac{1}{2I_2} - \dfrac{1}{2I_1}\right)$

$l = 0, 1, 2, \ldots$

$l : \text{fix} : m : -l, \ldots, l$

ب)

$$l=3 \longrightarrow E_{l=3,m} = \frac{6\hbar^2}{I_1} + \hbar^2\left(\frac{1}{2I_2} - \frac{1}{2I_1}\right)m^2$$

$$\begin{cases} m=\pm 3 \longrightarrow E = \dfrac{6\hbar^2}{I_1} + \dfrac{\hbar^2}{2}\left(\dfrac{1}{I_2} - \dfrac{1}{I_1}\right)9 \\ m=\pm 2 \longrightarrow E = \dfrac{6\hbar^2}{I_1} + \dfrac{\hbar^2}{2}\left(\dfrac{1}{I_2} - \dfrac{1}{I_1}\right)4 \\ m=\pm 1 \longrightarrow E = \dfrac{6\hbar^2}{I_1} + \dfrac{\hbar^2}{2}\left(\dfrac{1}{I_2} - \dfrac{1}{I_1}\right) \\ m=0 \longrightarrow E = \dfrac{6\hbar^2}{I_1} \end{cases}$$

ج)

$$I_1 = I_2 = I \longrightarrow E_l = \frac{\hbar^2 l(l+1)}{2I_1}$$

تبهگنی : $(2l+1)$

د)

$$\hbar\sqrt{l(l+1)} = \sqrt{56}\,\hbar \longrightarrow l=7$$

تبهگنی : ۱۵

سوال ۴ :

$$|\psi\rangle = \frac{1}{\sqrt{7}}|1,-1\rangle + A|1,0\rangle + \frac{\sqrt{2}}{\sqrt{7}}|1,1\rangle$$

$$1 = \frac{1}{7} + A^2 + \frac{2}{7} = \frac{3}{7} + A^2 \longrightarrow A^2 = \frac{4}{7} \longrightarrow A = \pm\frac{2}{\sqrt{7}}$$

$$\langle L^2\rangle = \langle\psi|\underbrace{L^2|\psi\rangle}_{2\hbar^2|\psi\rangle} = 2\hbar^2$$

$$\langle L_z\rangle = \langle\psi|\left(-\frac{\hbar}{\sqrt{7}}|1,-1\rangle + \frac{\hbar\sqrt{2}}{\sqrt{7}}|1,+1\rangle\right) = -\frac{\hbar}{7} + \frac{2\hbar}{7} = \frac{\hbar}{7}$$

$$\begin{cases} L_x = \dfrac{L_+ + L_-}{۲} \\ L_y = \dfrac{L_+ - L_-}{۲i} \end{cases}$$

$$\langle \psi | L_\pm | \psi \rangle = A \hbar \frac{\sqrt{۲}}{\sqrt{۷}} (1+\sqrt{۲})$$

$$A = \frac{۲}{\sqrt{۷}} \longrightarrow \langle L_\pm \rangle = \hbar \frac{۲\sqrt{۲}}{۷}(1+\sqrt{۲})$$

$$A = -\frac{۲}{\sqrt{۷}} \longrightarrow \langle L_\pm \rangle = -\hbar \frac{۲\sqrt{۲}}{۷}(1+\sqrt{۲})$$

$$\langle L_x \rangle = \frac{\langle L_+ \rangle + \langle L_- \rangle}{۲} = \langle L_\pm \rangle = \pm \hbar \frac{۲\sqrt{۲}}{۷} (1+\sqrt{۲})$$

$$\langle L_y \rangle = \frac{\langle L_+ \rangle - \langle L_- \rangle}{۲i} = ۰$$

12.

$$P(l_z = \hbar) = \frac{۲}{۷} = |\langle 1,1 | \psi \rangle|^۲$$

$$L_+^۲ |\psi\rangle = \frac{1}{\sqrt{۷}} \sqrt{l(l+1)-m(m+1)}\Big|_{m=-1} \sqrt{l(l+1)-m(m+1)}\Big|_{m=۰} |1,1\rangle \qquad ()$$

$$= \frac{۲}{\sqrt{۷}} |1,1\rangle$$

$$\langle 1,m | L_+^۲ | \psi \rangle = \frac{۲}{\sqrt{۷}} \delta_{m,1}$$

$$L_-^۲ |\psi\rangle = \frac{\sqrt{۲}}{\sqrt{۷}} \sqrt{l(l+1)-m(m-1)}\Big|_{m=1} \sqrt{l(l+1)-m(m-1)}\Big|_{m=۰} |1,-1\rangle$$

$$= \frac{۲\sqrt{۲}}{\sqrt{۷}} |1,-1\rangle$$

$$\langle 1,m | L_-^۲ | \psi \rangle = \frac{۲\sqrt{۲}}{۷} \delta_{m,-1}$$

سوال ۵ :

الف)

$$S = \frac{3}{2} \quad , \quad H = \frac{\varepsilon_0}{\hbar^2}\left(S_x^2 - S_y^2\right) - \frac{\varepsilon_0}{\hbar^2} S_z^2$$

برای نمایش ماتریسی پایه فضا را ویژه بردارهای S_z انتخاب می‌کنیم به ترتیب زیر :

$$\left\{ |s=3/2, m=3/2\rangle , |s=3/2, m=1/2\rangle , |s=3/2, m=-1/2\rangle , |s=3/2, m_s=-3/2\rangle \right\}$$

$$\langle s, m' | \hat{S}_z | s, m\rangle = m\frac{\hbar}{2}\,\delta_{mm'} \longrightarrow \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 3 & & & \\ & 1 & & \\ & & -1 & \\ & & & -3 \end{pmatrix}$$

$$\longrightarrow \hat{S}_z^2 = \frac{\hbar^2}{4}\begin{pmatrix} 9 & & & \\ & 1 & & \\ & & 1 & \\ & & & 9 \end{pmatrix}$$

$$\langle s, m' | \hat{S}_\pm | s, m\rangle = \hbar\sqrt{s(s+1) - m(m\pm 1)}\;\delta_{m', m\pm 1}$$

$$\hat{S}_+ = \hbar\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \hat{S}_- = \hbar\begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

$$S_\pm = S_x \pm i S_y \longrightarrow \begin{cases} S_x = \dfrac{S_+ + S_-}{2} \\[2mm] S_y = \dfrac{S_+ - S_-}{2i} \end{cases}$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & & \\ \sqrt{3} & 0 & 2 & \\ & 2 & 0 & \sqrt{3} \\ & & \sqrt{3} & 0 \end{pmatrix} \qquad S_y = \frac{i\hbar}{2}\begin{pmatrix} 0 & -\sqrt{3} & & \\ \sqrt{3} & 0 & -2 & \\ & 2 & 0 & -\sqrt{3} \\ & & \sqrt{3} & 0 \end{pmatrix}$$

$$S_x^2 = \frac{\hbar^2}{4}\begin{pmatrix} 3 & 0 & 2\sqrt{3} & 0 \\ 0 & 7 & 0 & 2\sqrt{3} \\ 2\sqrt{3} & 0 & 7 & 0 \\ 0 & 2\sqrt{3} & 0 & 3 \end{pmatrix} \qquad S_y^2 = \frac{\hbar^2}{4}\begin{pmatrix} 3 & 0 & -2\sqrt{3} & 0 \\ 0 & 7 & 0 & -2\sqrt{3} \\ -2\sqrt{3} & 0 & 7 & 0 \\ 0 & -2\sqrt{3} & 0 & 3 \end{pmatrix}$$

$$\longrightarrow H = \frac{\varepsilon_0}{4}\begin{pmatrix} -9 & 0 & 4\sqrt{3} & 0 \\ 0 & -1 & 0 & 4\sqrt{3} \\ 4\sqrt{3} & 0 & -1 & 0 \\ 0 & 4\sqrt{3} & 0 & -9 \end{pmatrix}$$

$$\det(H - EI) = 0 \longrightarrow E = \frac{\varepsilon_0}{4}(-13),\ \frac{\varepsilon_0}{4}(-13)$$

$$\frac{\varepsilon_0}{4}(3),\ \frac{\varepsilon_0}{4}(3)$$

H : دو ویژه مقدار با واگنی از مرتبه ۲ دارد .

ب)

$$E = \frac{3\varepsilon_0}{4} : \qquad H\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \frac{3\varepsilon_0}{4}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} -9a + 4\sqrt{3}\,c \\ -b + 4\sqrt{3}\,d \\ 4\sqrt{3}\,a - c \\ 4\sqrt{3}\,b - 9d \end{pmatrix} = 3\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \longrightarrow \begin{cases} c = \sqrt{3}\,a \\ b = \sqrt{3}\,d \end{cases}$$

$$a^2 + b^2 + c^2 + d^2 = 1$$

$$4a^2 + 4d^2 = 1$$

دو بردار عمود برهم دلخواه باید در این زیرفضا انتخاب کنیم .

یک گزینه :

$$\begin{cases} a = 0 \\ , \\ d = \frac{1}{2} \end{cases} \quad , \quad \begin{cases} a = 1/2 \\ , \\ d = 0 \end{cases}$$

$$\downarrow$$

$$|E=3,(1)\rangle = \begin{pmatrix} 0 \\ \sqrt{3}/2 \\ 0 \\ 1/2 \end{pmatrix} \qquad |E=3,(2)\rangle = \begin{pmatrix} 1/2 \\ 0 \\ \sqrt{3}/2 \\ 0 \end{pmatrix}$$

$$E = -\frac{13\varepsilon_0}{4} \longrightarrow \begin{pmatrix} -9a + 4\sqrt{3}\,c \\ -b + 4\sqrt{3}\,d \\ 4\sqrt{3}\,a - c \\ 4\sqrt{3}\,b - 9d \end{pmatrix} = -13\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\longrightarrow \begin{cases} a = -\sqrt{3}\, c \\ d = -\sqrt{3}\, b \end{cases} \qquad \begin{aligned} a^2+b^2+c^2+d^2 &= 1 \\ 4c^2+4b^2 &= 1 \end{aligned}$$

با محاسبه :

$$\begin{cases} c = 0 \\ b = 1/2 \end{cases} \longrightarrow |E=-12,(1)\rangle = \begin{pmatrix} 0 \\ 1/2 \\ 0 \\ -\sqrt{3}/2 \end{pmatrix}$$

$$\begin{cases} c = 1/2 \\ b = 0 \end{cases} \longrightarrow |E=12,(2)\rangle = \begin{pmatrix} -\sqrt{3}/2 \\ 0 \\ 1/2 \\ 0 \end{pmatrix}$$

← در نهایت هر ۴ بردار دو به دو بر یکدیگر عمود اند .

ج)

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1/2 \\ 0 \\ \sqrt{3}/2 \\ 0 \end{pmatrix} - \frac{\sqrt{3}}{2}\begin{pmatrix} -\sqrt{3}/2 \\ 0 \\ 1/2 \\ 0 \end{pmatrix} = \frac{1}{2}|E=4,(2)\rangle - \frac{\sqrt{3}}{2}|E=-12,(2)\rangle$$

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle = e^{-i\hat{H}t/\hbar}\left(\frac{1}{2}|E=4,(2)\rangle - \sqrt{3}/2\,|E=-12,(2)\rangle\right)$$

$$= \frac{1}{2}\underbrace{e^{-i\hat{H}t/\hbar}|E=4,(2)\rangle}_{e^{-i\left(\frac{\varepsilon_0}{4}4\right)t/\hbar}|E=4,(2)\rangle} - \sqrt{3}/2\,\underbrace{e^{-iHt/\hbar}|E=-12,(2)\rangle}_{e^{-i\left(\frac{\varepsilon_0}{4}(-12)\right)t/\hbar}|E=-12,(2)\rangle}$$

$$= \frac{1}{2}e^{-i\frac{4\varepsilon_0}{4}t/\hbar}\begin{pmatrix} 1/2 \\ 0 \\ \sqrt{3}/2 \\ 0 \end{pmatrix} - \sqrt{3}/2\, e^{+i\frac{12}{4}\varepsilon_0 t/\hbar}\begin{pmatrix} -\sqrt{3}/2 \\ 0 \\ 1/2 \\ 0 \end{pmatrix}$$

$$|\psi(t)\rangle = \frac{1}{4}\begin{pmatrix} e^{-i\frac{4}{4}\varepsilon_0 t/\hbar} + 3\, e^{i\frac{12}{4}\frac{\varepsilon_0 t}{\hbar}} \\ 0 \\ \sqrt{3}\left(e^{-i\frac{4}{4}\frac{\varepsilon_0 t}{\hbar}} - e^{i\frac{12}{4}\varepsilon_0 t/\hbar}\right) \end{pmatrix}$$

سوال ششم :

$$\psi(\theta,\varphi) = 3\sin\theta\cos\theta\, e^{i\varphi} - 2(1-\cos^2\theta)\, e^{2i\varphi}$$

الف)

$$Y_{2,1} = -\frac{1}{2}\left(\frac{15}{2\pi}\right)^{1/2} \sin\theta\cos\theta\, e^{i\varphi}$$

$$Y_{2,2} = \frac{1}{4}\left(\frac{15}{2\pi}\right)^{1/2} \sin^2\theta\, e^{2i\varphi}$$

$$\longrightarrow \psi(\theta,\varphi) = 3\left(-2\left(\frac{2\pi}{15}\right)^{1/2} Y_{2,1}(\theta,\varphi)\right) - 2\left(4\left(\frac{2\pi}{15}\right)^{1/2} Y_{2,2}(\theta,\varphi)\right)$$

$$\psi(\theta,\varphi) = -2\sqrt{\frac{2\pi}{15}}\left(3Y_{2,1}(\theta,\varphi) + 4Y_{2,2}(\theta,\varphi)\right)$$

ب)

$$L^2\psi = \hbar^2 l(l+1)\Big|_{l=2}\psi$$

ویژه مقدار L^2 است با ویژه مقدار $6\hbar^2$

پس ویژه مقدار L_z نیست.

ج)

$$\psi(\theta,\varphi) = \frac{3}{5}Y_{2,1} + \frac{4}{5}Y_{2,2}$$

نرمال شده

$$P(L_z = 2\hbar) = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$